

**Day 1 - Session 2a**  
**Theoretical Highlights in Fiscal**  
**Incidence Analysis**  
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**Learning Event on the Commitment to**  
**Equity Methodology**  
**CEQ Institute and The World Bank**  
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Today's presentation is based on the theory chapter:

- Enami, A., N. Lustig and R. Aranda. “Analytical Foundations: Measuring the Redistributive Impact of Taxes and Transfers ” Chapter 6 in Lustig (editor) *Commitment to Equity Handbook. A Guide to Estimating the Impact of Fiscal Policy on Inequality and Poverty*, Tulane University, Fall 2016.

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# Fiscal Policy and Inequality

## Four Key Questions

- Does the net fiscal system decrease inequality?
  - Equation for the fiscal system
- Is a particular tax or transfer equalizing or unequalizing?
  - Sign of marginal contribution
- What is the contribution of a particular tax or transfer (or any combination of them) to the change in inequality?
  - Size of the marginal contribution
- What is the inequality impact if one increases the size of a tax (transfer) or its progressivity?
  - Derivative of the marginal contribution

# Chapter Outline

- **Fiscal Redistribution: Single and Multiple Interventions (Chapter 6)**
- Allowing for Reranking (Chapter 7)
- Allowing for No Dominance
- Allowing for Different Original Distributions
- Different Inequality Measures
- Poverty



# Assumptions for Now

- **No reranking:** the ordering of individuals in the post-fiscal state is the same as in the pre-fiscal state: i.e., no swapping of places
- **Dominance:** pre-fiscal and post-fiscal Lorenz curves do not cross (and the difference is statistically significant)
- **Same pre-fiscal (original) income distribution:** rules out comparisons of redistributive of fiscal systems across countries and over-time

# Key questions addressed for the following cases

- Single intervention system:
  - Tax OR
  - Transfer
- Multiple interventions system
  - One tax and one transfer
  - $n$  taxes and  $m$  transfers
- Lambert's conundrum and the startling consequences of path dependency

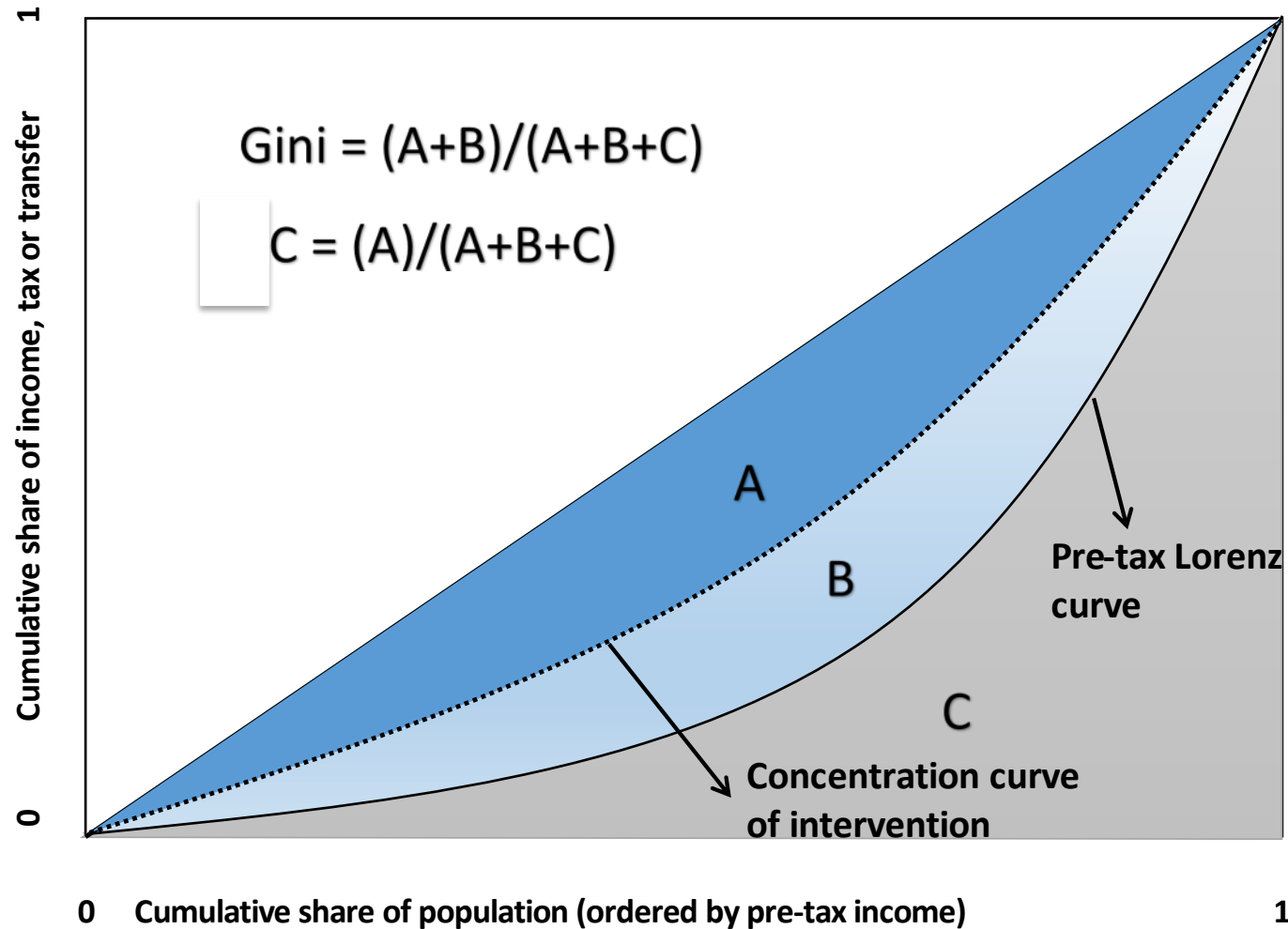
# **Fiscal System with a Single Intervention**

# Single Intervention

- *Single* can mean that all the taxes are added into a single category (same for transfers)
- Progressivity measures
  - Concentration curve
  - Concentration coefficient
  - Kakwani Index



# Concentration Coefficient: C



# Kakwani Index

➤ Progressive Tax:  $\prod_T^K = C_t - G_x > 0$

➤ Proportional Tax:  $\prod_T^K = C_t - G_x = 0$

➤ Regressive Tax:  $\prod_T^K = C_t - G_x < 0$

# Impact on Inequality Depends On...

- Progressivity of a tax (transfer)
- Size of the tax (transfer), where size equals the total tax (transfer) divided by total pre-tax (pre-transfer) income
  - A large regressive tax can be more equalizing than a small progressive one

# **Fiscal Policy and Inequality**

## **Four Key Questions**

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfer (or any combination of them) to the change in inequality?
- What is the inequality impact if one increases the size of a tax (transfer) or its progressivity?



# Progressivity vs. Size of Intervention:

## A System with Only One Tax

- In a system with only one tax:

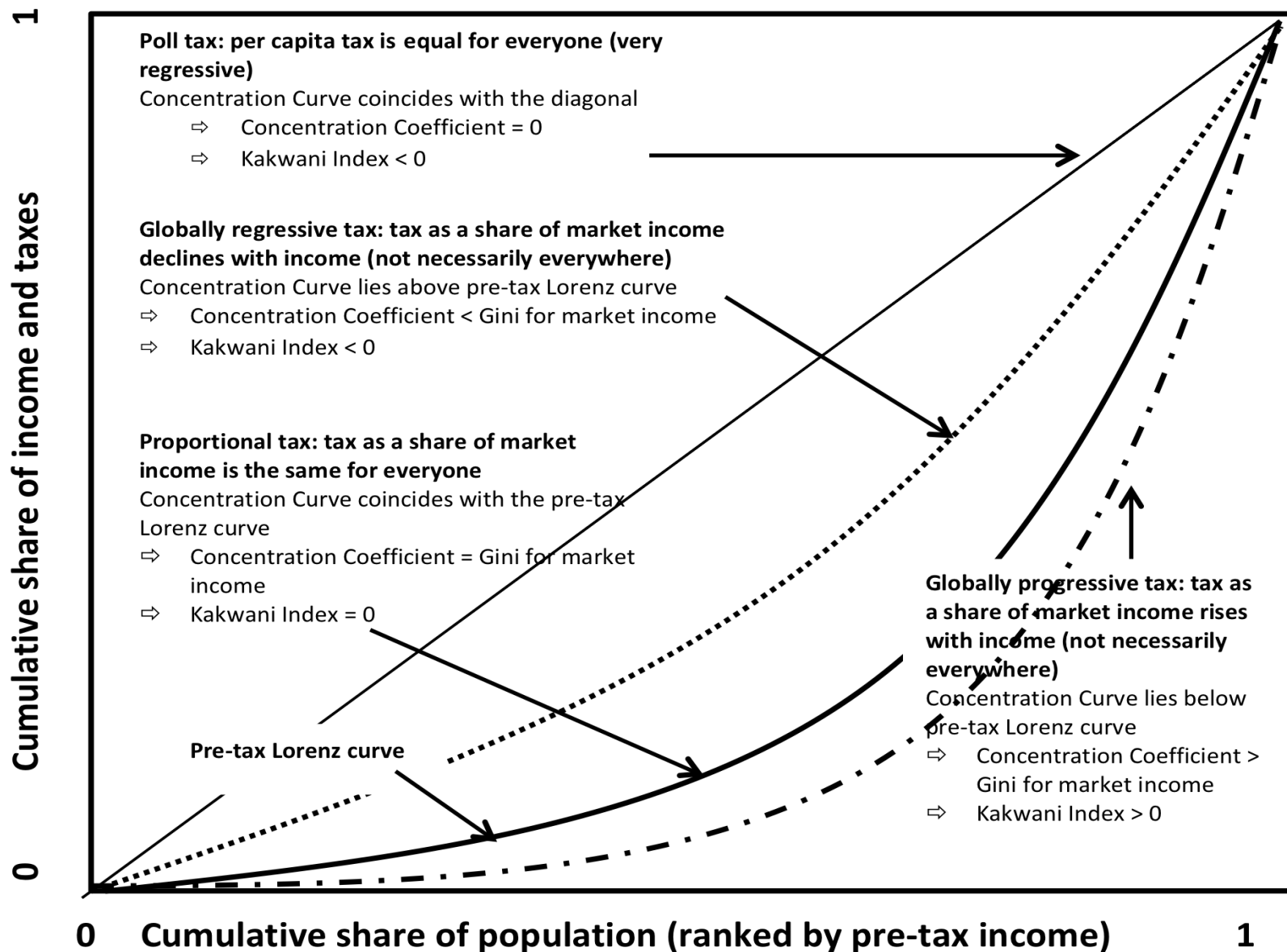
$$RE_T = \frac{g}{1-g} \Pi_T^K$$

- Getting the partial derivatives:

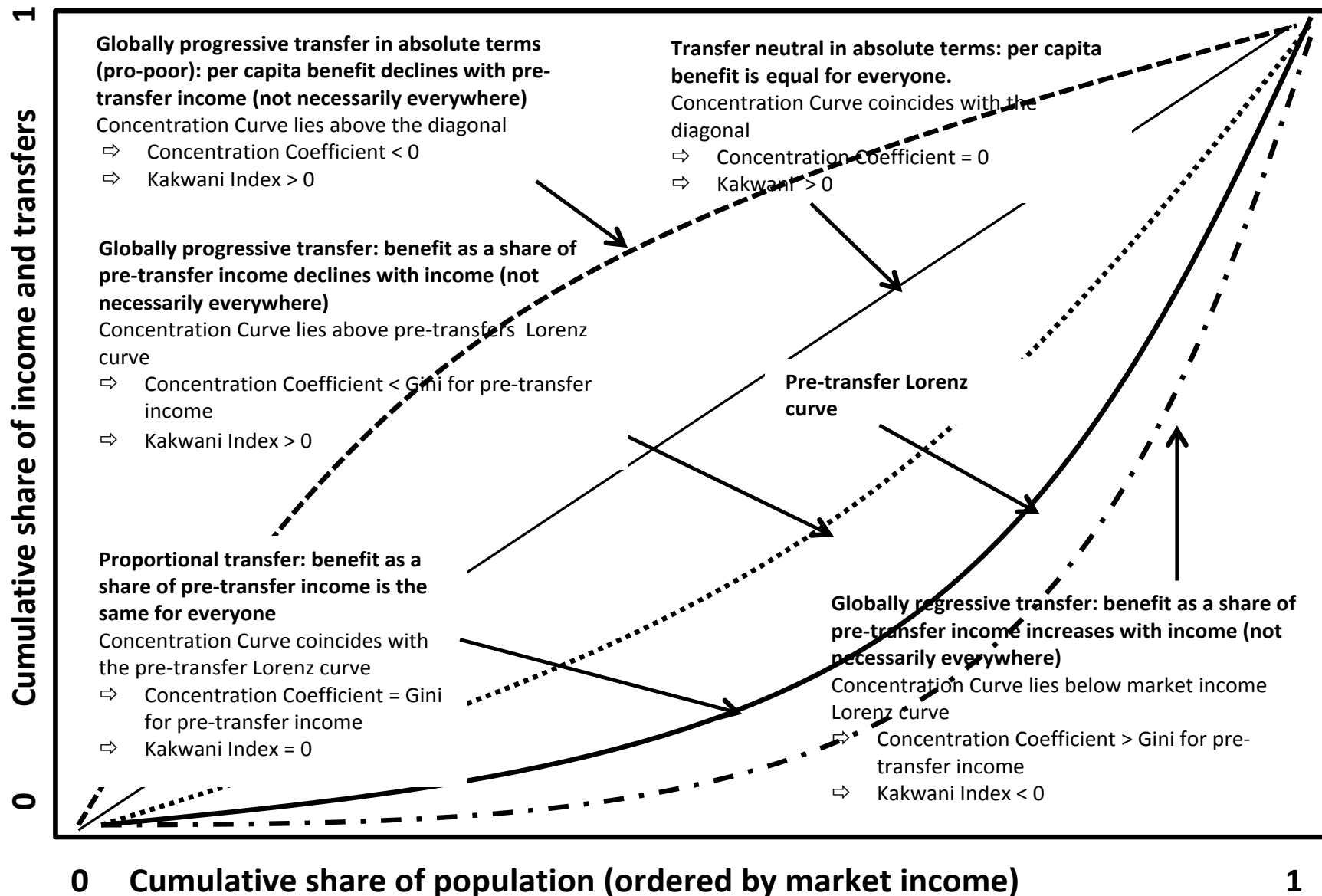
$$\frac{\partial RE_T}{\partial g} = \frac{1}{(1-g)^2} \Pi_T^K$$

$$\frac{\partial RE_T}{\partial \Pi_T^K} = \frac{g}{1-g}$$

## Progressivity of Taxes: A Diagrammatic Representation



# Progressivity of Transfers: A Diagrammatic Representation



# **Fiscal System with Multiple Interventions**



# **Fiscal Policy and Inequality**

## **Four Key Questions**

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfer (or any combination of them) to the change in inequality?
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## Does the net fiscal system decrease inequality?

Let's define the Redistributive Effect of the net fiscal system as

$$RE_N = G_x - G_N$$

Where  $G_x$  *and*  $G_N$  are the pre-tax-pre-transfer Gini coefficient and post-tax-post-transfer Gini, respectively

# Does the net fiscal system decrease inequality?

From Lambert (2001), we know that  $RE_N$  is equal to the weighted sum of the redistributive effect of taxes and transfers

$$RE_N = \frac{(1 - g)RE_t + (1 + b)RE_B}{1 - g + b}$$

Where

- $RE_t$  and  $RE_B$  are the Redistributive Effect of the tax and the transfer, respectively
- $g$  and  $b$ : size of tax and transfer, respectively.  
That is, total taxes and total transfers divided by total pre-tax and pre-transfer income, respectively

# Does the net fiscal system decrease inequality?

For the net fiscal system to be equalizing:

$$RE_N = \frac{(1-g)RE_t + (1+b)RE_B}{1-g+b} > 0$$

Condition 1:

$$\rightarrow RE_t > -\frac{(1+b)}{(1-g)} RE_B$$

# Does the net fiscal system decrease inequality?

		Transfer		
		Regressive $\rho_B^K < 0$	Neutral $\rho_B^K = 0$	Progressive $\rho_B^K > 0$
Tax	Regressive $\Pi_T^K < 0$	Always Unequalizing	Always Unequalizing	Equalizing if and only if Condition 1 holds
	Neutral $\Pi_T^K = 0$	Always Unequalizing	No Change in Equality	Always Equalizing
	Progressive $\Pi_T^K > 0$	Equalizing if and only if Condition 1 holds	Always Equalizing	Always Equalizing

Condition 1:

$$\rightarrow RE_t > -\frac{(1+b)}{(1-g)} RE_B$$

- The above result is well-known in the literature:
  - A fiscal system with a regressive tax can be equalizing as long as transfers are progressive and the condition above is fulfilled
  - A fiscal system with a regressive tax that collects more revenues than a less regressive one may be more equalizing
- However, Lambert's equation has more fundamental implications

# **Fiscal Policy and Inequality**

## **Four Key Questions**

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfer (or any combination of them) to the change in inequality?
- What is the inequality impact if one increases the size of a tax (transfer) or its progressivity?

# Is a particular tax or transfer equalizing?

- If there is a single intervention in the system, any of the progressivity measures discussed earlier will give an unambiguous answer
- If there is a tax **and** a transfer, then this is no longer the case
  - A regressive tax can be equalizing in the sense that the reduction in inequality can be larger with the tax than without it



# Lambert's Conundrum

	1	2	3	4	Total
Original Income $x$	10	20	30	40	100
Tax $t$	6	9	12	15	42
Transfer $B$	21	14	7	0	42
Net Income $N$	25	25	25	25	100

Source: Lambert, 2001, Table 11.1, p. 278

# Lambert's Conundrum

- The Redistributive Effect of the tax only in this example is equal to -0.05, highlighting its regressivity
- The Redistributive Effect of the transfer is equal to 0.19
- Yet, the Redistributive Effect of the net fiscal system is 0.25, higher than the effect without the taxes!

# Lambert's Conundrum

	1	2	3	4	Total
Original Income $x$	10	20	30	40	100
Transfer $B$	21	14	7	0	42
Post-Transfer Income	31	34	37	40	142
Tax $t$	6	9	12	15	42
Net Income $N$	25	25	25	25	100

Source: Lambert, 2001, Table 11.1, p. 278

# Lambert's Conundrum

## Path Dependency

- If a tax is regressive vis-à-vis the original income but progressive with respect to the less unequally distributed post-transfer income
- Regressive taxes *can* exert an equalizing effect over and above the effect of progressive transfers
- Note that institutional path dependency is not the same as mathematical path dependency

# When could a regressive tax exert an equalizing force?

For the reduction in inequality to be higher with the tax than without it, the following condition must hold:

$$RE_N = \frac{(1 - g)RE_t + (1 + b)RE_B}{1 - g + b} > RE_B$$

Condition 2

$$\rightarrow RE_t > -\frac{(g)}{(1 - g)} RE_B$$

# Is a tax equalizing?

## Answer for a system with a tax and a transfer

		System with a Transfer that is		
		Regressive $\rho_B^K < 0$	Neutral $\rho_B^K = 0$	Progressive $\rho_B^K > 0$
Adding a Tax that is	Regressive $\Pi_T^K < 0$	Always More Unequalizing	Always Unequalizing	More Equalizing only if Condition 2 holds
	Neutral $\Pi_T^K = 0$	Always More Unequalizing	No Change in Inequality	Always More Equalizing
	Progressive $\Pi_T^K > 0$	More Equalizing only if Condition 2 holds	Always Equalizing	Always More Equalizing

Condition 2

$$\rightarrow RE_t > -\frac{(g)}{(1-g)} RE_B$$

# Equalizing Regressive Taxes Exist in Real Life

- The US and the UK had regressive equalizing taxes in the past (O'Higgins & Ruggles, 1981 and Ruggles & O'Higgins, 1981)
- Chile's 1996 fiscal system had equalizing regressive taxes (Engel et al., 1999)
  - Redistributive Effect of Net Fiscal System (taxes and transfers together = 0.0583 (decline in Gini points)
  - Redistributive Effect of System with Taxes only = - 0.0076
  - Redistributive Effect of System with Transfers but without Taxes = 0.0574
- Note that  $0.0583 > 0.0574$
- CEQs for Chile 2013 and South Africa 2010 also show that regressive consumption taxes are equalizing

# Generalizing the result to $n$ taxes and $m$ transfers

## Is a particular tax or transfer equalizing?

- The results shown above can be generalized to  $n$  taxes and  $m$  transfers (in chapter but not presented here)
- Note that the results do not require for the size of total taxes and total transfers to be the same (see conditions 1 and 2 above)



# Path Dependency Underscores the Importance of the Analysis Being Comprehensive

- Obvious reason
  - To capture the full effect of the net fiscal system
- More subtle but fundamental reason
  - Assessing the progressivity of a tax or a transfer in isolation can give the wrong answer to the question: Is the tax or the transfer equalizing?
  - Think of the example of Chile and South Africa just shown above

# Calculating the Marginal Contribution of a Tax

The marginal contribution of a tax is defined as

$$MC_t = G_{x+B} - G_{x+B-t}$$

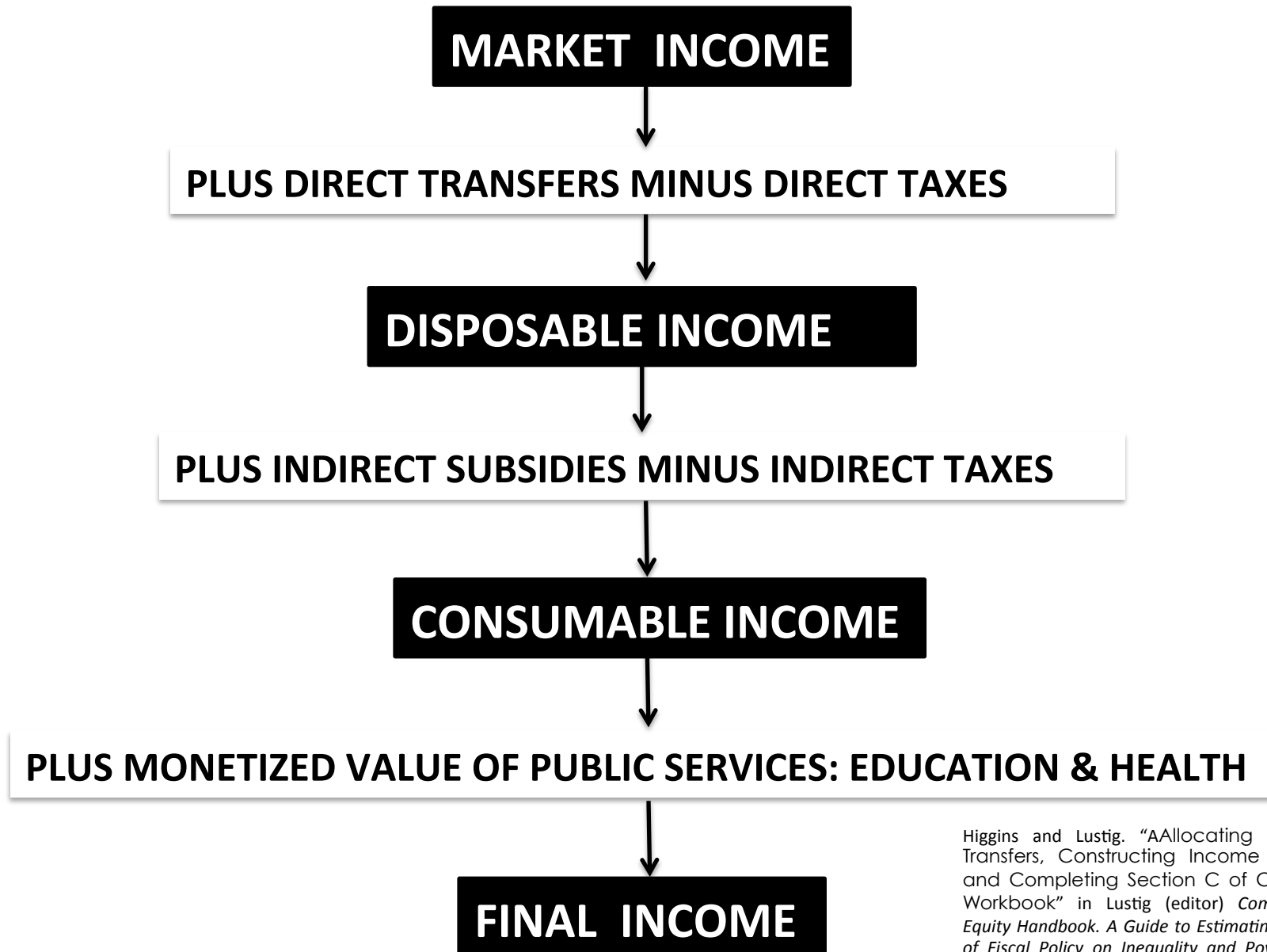
Where  $G_{x+B}$ ,  $G_{x+B-t}$  and are the Gini coefficient of income with the transfer but **without** the tax and the Gini coefficient with the transfer and **with** the tax, respectively

If  $MC_t > 0$ , remember, the tax is equalizing



# Which fiscal instruments are equalizing and which are not?

- Rely on the sign of the “marginal contribution”
- The marginal contribution equals the difference in the reduction in inequality observed without the fiscal instrument of interest (and all the others in place) and with it (and all the others in place)
  - Positive means it is equalizing
  - Negative means it is unequalizing
  - Zero means it leaves inequality unchanged
  - The following tables use the change in Gini points to measure fiscal redistribution



Higgins and Lustig. "Allocating Taxes and Transfers, Constructing Income Concepts, and Completing Section C of CEQ Master Workbook" in Lustig (editor) *Commitment to Equity Handbook. A Guide to Estimating the Impact of Fiscal Policy on Inequality and Poverty*, Tulane University, Fall 2016.

# Marginal Contributions (contributory pensions as deferred income) – Low-income and lower-middle-income economies (In Gini points)

The unequalizing effects appear in red font.

Source: Lustig (2016)	Low-income Economies	Lower-middle-income economies						
	Tanzania (2011)	Armenia (2011)	Bolivia (2009)	El Salvador (2011)	Georgia (2013)	Ghana (2013)	Indonesia (2012)	Sri Lanka (2010)
Redistributive effect (from Gini market income plus pensions to <u>consumable</u> income)	0.0400	0.0950	0.0031	0.0101	0.0961	0.0138	0.004	0.0115
<b>Marginal contribution</b>								
Direct taxes	0.0023	0.019	--	0.0037	0.0207	-0.0048	0.000	0.0025
Direct transfers	0.0011	0.101	0.0127	0.0064	0.1131	0.0013	0.004	0.0044
Indirect taxes	0.0181	-0.001	-0.0093	-0.0006	-0.0179	0.0010	-0.003	-0.0003
Indirect subsidies	-0.0035	0.000	0.0009	0.0014	0.0005	-0.0007	0.003	0.0057

# Marginal Contributions (contributory pensions as deferred income) –

## Upper-middle-income and high-income economies

### (In Gini points)

The unequalizing effects appear in red font.

Source: Lustig (2016)	Upper-middle-income Economies								High-income Economies		
	Brazil (2009)	Colombia (2010)	Costa Rica (2010)	Ecuador (2011)	Jordan (2010)	Mexico (2010)	Peru (2009)	South Africa (2010)	Chile (2013)	Russia (2010)	Uruguay (2009)
Redistributive effect (from Gini market income plus pensions to <u>consumable</u> income)	0.0354	0.0116	0.0226	0.0333	0.0161	0.0308	0.0151	0.0766	0.0295	0.028	0.0337
<b>Marginal contribution</b>											
Direct taxes	0.0130	-0.0008	0.0092	0.0044	0.0076	0.0140	0.0060	0.0327	0.0119	0.011	0.0167
Direct transfers	0.0216	0.0065	0.0135	0.0178	0.0055	0.0113	0.0048	0.0672	0.0225	0.024	0.0247
Indirect taxes	0.0032	-0.0020	0.0029	0.0042	-0.0015	0.0027	0.0052	-0.0002	0.0002	-0.004	-0.0034
Indirect subsidies	0.0007	0.0055	--	0.0078	0.0046	0.0047	--	--	0.0028	0.000	--

# In sum...

- Direct taxes are equalizing except in Colombia and Ghana (surprised?).
- Direct transfers are always equalizing (pew!).
- Indirect taxes are more often than not unequalizing, but they are equalizing in several countries: Brazil, Chile, Costa Rica, Ecuador, Ghana, Mexico, Peru, and Tanzania (surprised?).
- Indirect subsidies are more often than not equalizing (surprised?), except in Armenia, Ghana, and Tanzania.

# Fiscal Policy and Inequality

## Four Key Questions

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- Is a particular tax or transfer equalizing or unequalizing?
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# What is the contribution of a particular tax or transfer to the change in inequality?

- Sequential method
  - May give the wrong answer to the “without vs. with comparison” because it ignores path dependency
- **Marginal contribution method (same for poverty)**
  - Gives correct answer to the “without vs. with comparison” but does not fulfill the principle of aggregation: i.e., the sum of the marginal contributions will not equal the total change in inequality (except by coincidence)
- Average Contribution with all possible paths considered (Shapley value)
  - Fulfills the principle of aggregation, takes care of path dependency but the sign may be different from the marginal contribution => problematic?

# Sequential vs. Marginal Contribution

## Why the sequential method can be misleading

Chile's 1996 fiscal system (Engel et al., 1999)

- Sequential contribution method: -0.0076
- Marginal contribution method: 0.009

# **Fiscal Policy and Inequality**

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# Progressivity vs. Size of Intervention:

## A System with One Tax and One Transfer

- In a system with one tax and one transfer:

$$MC_T = G_{X+B} - G_{X-T+B} = \dots = \frac{g \Pi_T^K + b \rho_B^K}{1 - g + b} - \frac{b}{1 + b} \rho_B^K$$

- Getting the partial derivatives:

$$\frac{\partial MC_T}{\partial g} = \frac{(1 + b) \Pi_T^K + b \rho_B^K}{(1 - g + b)^2}$$

$$\frac{\partial MC_T}{\partial \Pi_T^K} = \frac{g}{1 - g + b}$$

# References

- Duclos, Jean-Yves and Abdelkrim Araar. 2007. *Poverty and Equity: Measurement, Policy and Estimation with DAD* (Vol. 2). Springer. Chapters 7 and 8. (available online)
- Lambert, Peter J. (2001). *The Distribution and Redistribution of Income: A Mathematical Analysis*. Manchester University Press. Third Edition. Chapter 11. (not available online)

# Thank you!