Session 2
The Analytics of Fiscal Redistribution

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Fiscal Policy and Inequality
Three Key Questions

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfers (or any combination of them) to the change in inequality?
Fiscal Policy and Poverty
Three Key Questions

- Does the net fiscal system decrease poverty?
- Does the net fiscal system make the poor poorer?
- What is the contribution of a particular tax or transfers (or any combination of them) to the change in poverty?
Key questions will be addressed for two main cases

- Single-intervention system:
  - Tax
  - Transfer

- Multiple-interventions system
  - Lambert’s conundrum and the startling consequences of path dependency
Assumptions

- **No reranking:** the ordering of individuals in the post-fiscal state is the same as in the pre-fiscal state: i.e., no swapping of places

- **Dominance:** pre-fiscal and post-fiscal Lorenz curves do not cross (and the difference is statistically significant)

- **Same pre-fiscal (original) income distribution:** rules out comparisons of redistributive or poverty reducing capacity of fiscal systems across countries and over-time
Fiscal Incidence Analysis

\[ Y_h = I_h - \sum_i T_i S_{ih} + \sum_j B_j S_{jh} \]

- Income after taxes and transfers
- Taxes
- Transfers

Income before taxes and transfers
Share of tax \( i \) paid by unit \( h \)
Share of transfer \( j \) received by unit \( h \)
FISCAL SYSTEM WITH A SINGLE INTERVENTION
Single Intervention: Tax

• Progressivity measures

➢ Concentration curve
➢ Concentration coefficient
➢ Kakwani Index
Concentration Curve Progressive Tax

Post-tax Lorenz curve

Distribution became more equal

Pre-tax Lorenz curve

Concentration curve of a progressive tax

Cumulative share of income and taxes

Cumulative share of population (ranked by pre-tax income)
Concentration Curve

Concentration curve of a regressive tax

Pre-tax Lorenz curve

Post-tax Lorenz curve

Distribution became more unequal

Cumulative share of population (ranked by pre-tax income)
Concentration Coefficient: CC

Vertical Axis
Cumulative proportion of income, tax or transfer

\[ \text{Gini} = \frac{A}{(A+B)} \]

\[ \text{CC} = \frac{A}{(A+B)} \]

Diagram showing the cumulative proportion of population ranked by income and the Gini coefficient/concentration index.
Kakwani Index: Tax

The Kakwani index of progressivity of a tax $t$ is defined as:

$$K_t = CC_t - G_x$$

Where:

- $G_x$ is the Gini coefficient of pre-tax income
- $CC_t$ is the concentration coefficient of the tax $t$
Kakwani Index

- Progressive Tax: \( K_t = CC_t - G_x > 0 \)
- Proportional Tax: \( K_t = CC_t - G_x = 0 \)
- Regressive Tax: \( K_t = CC_t - G_x < 0 \)
Progressivity of Taxes: A Diagrammatic Representation

1
Poll tax: per capita tax is equal for everyone (very regressive)
Concentration Curve coincides with the diagonal
⇒ Concentration Coefficient = 0
⇒ Kakwani Index < 0

Globally regressive tax: tax as a share of market income declines with income (not necessarily everywhere)
Concentration Curve lies above pre-tax Lorenz curve
⇒ Concentration Coefficient < Gini for market income
⇒ Kakwani Index < 0

Proportional tax: tax as a share of market income is the same for everyone
Concentration Curve coincides with the pre-tax Lorenz curve
⇒ Concentration Coefficient = Gini for market income
⇒ Kakwani Index = 0

Globally progressive tax: tax as a share of market income rises with income (not necessarily everywhere)
Concentration Curve lies below pre-tax Lorenz curve
⇒ Concentration Coefficient > Gini for market income
⇒ Kakwani Index > 0

Cumulative share of income and taxes
0
0
Cumulative share of population (ranked by pre-tax income)
Conclusion

In a world with just a *single* tax

- A necessary and sufficient condition for a tax to be equalizing is to have a positive Kakwani index

- A necessary and sufficient condition for a tax to be unequalizing is to have a negative Kakwani index
Progressivity: Everywhere vs. Global

- A tax can be progressive and equalizing even if it is not progressive everywhere as long as it is globally progressive.

- The toy example below illustrates this point.
# Toy Example: An Everywhere vs. Globally Progressive Tax

## Everywhere Progressive Tax

<table>
<thead>
<tr>
<th>Population</th>
<th>Pre-tax Income</th>
<th>Lorenz Curve Pre-tax</th>
<th>Tax Rate Everywhere Progressive Tax</th>
<th>Tax paid</th>
<th>Post-tax Income</th>
<th>Lorenz Curve Post-tax</th>
<th>Difference between post- and pre-tax Lorenz curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.00</td>
<td>10%</td>
<td>0%</td>
<td>$0.00</td>
<td>$10.00</td>
<td>13%</td>
<td>2.50%</td>
</tr>
<tr>
<td>2</td>
<td>$20.00</td>
<td>30%</td>
<td>10%</td>
<td>$2.00</td>
<td>$18.00</td>
<td>35%</td>
<td>5.00%</td>
</tr>
<tr>
<td>3</td>
<td>$30.00</td>
<td>60%</td>
<td>20%</td>
<td>$6.00</td>
<td>$24.00</td>
<td>65%</td>
<td>5.00%</td>
</tr>
<tr>
<td>4</td>
<td>$40.00</td>
<td>100%</td>
<td>30%</td>
<td>$12.00</td>
<td>$28.00</td>
<td>100%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5</td>
<td>$100.00</td>
<td>20%</td>
<td>$20.00</td>
<td>$80.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Globally Progressive Tax

<table>
<thead>
<tr>
<th>Population</th>
<th>Pre-tax Income</th>
<th>Lorenz Curve Pre-tax</th>
<th>Tax Rate Progressive Not Everywhere</th>
<th>Tax paid</th>
<th>Post-tax Income</th>
<th>Lorenz Curve Post-tax</th>
<th>Difference between post- and pre-tax Lorenz curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.00</td>
<td>10%</td>
<td>0%</td>
<td>$0.00</td>
<td>$10.00</td>
<td>13%</td>
<td>2.50%</td>
</tr>
<tr>
<td>2</td>
<td>$20.00</td>
<td>30%</td>
<td>10%</td>
<td>$2.00</td>
<td>$18.00</td>
<td>35%</td>
<td>5.00%</td>
</tr>
<tr>
<td>3</td>
<td>$30.00</td>
<td>60%</td>
<td>0%</td>
<td>$0.00</td>
<td>$30.00</td>
<td>73%</td>
<td>12.50%</td>
</tr>
<tr>
<td>4</td>
<td>$40.00</td>
<td>100%</td>
<td>45%</td>
<td>$18.00</td>
<td>$22.00</td>
<td>100%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5</td>
<td>$100.00</td>
<td>20%</td>
<td>$20.00</td>
<td>$80.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Single Intervention: Transfer

- Progressivity measures
  - Concentration curve
  - Concentration coefficient
  - Kakwani Index
Kakwani Index: Transfer

The Kakwani index of progressivity of a transfer $B$ is defined as:

$$K_B = G_x - CC_B$$

Where:

- $G_x$ is the Gini coefficient of pre-tax income
- $CC_B$ is the concentration coefficient of the transfer $B$

Note that the Gini coefficient and the concentration coefficient are in reversed order from the Kakwani index for a tax
Progressivity of Transfers: A Diagrammatic Representation

- **Globally progressive transfer in absolute terms (pro-poor):** per capita benefit declines with pre-transfer income (not necessarily everywhere)
  - Concentration Curve lies above the diagonal
  - Concentration Coefficient < 0
  - Kakwani Index > 0

- **Transfer neutral in absolute terms:** per capita benefit is equal for everyone.
  - Concentration Curve coincides with the diagonal
  - Concentration Coefficient = 0
  - Kakwani > 0

- **Globally progressive transfer:** benefit as a share of pre-transfer income declines with income (not necessarily everywhere)
  - Concentration Curve lies above pre-transfer Lorenz curve
  - Concentration Coefficient < Gini for pre-transfer income
  - Kakwani Index > 0

- **Proportional transfer:** benefit as a share of pre-transfer income is the same for everyone
  - Concentration Curve coincides with the pre-transfer Lorenz curve
  - Concentration Coefficient = Gini for pre-transfer income
  - Kakwani Index = 0

- **Globally regressive transfer:** benefit as a share of pre-transfer income increases with income (not necessarily everywhere)
  - Concentration Curve lies below market income Lorenz curve
  - Concentration Coefficient > Gini for pre-transfer income
  - Kakwani Index < 0
CEQ Logo: Can you guess what it symbolizes?
Impact on Inequality Depends On...

- Progressivity of the tax or the transfer
- Level of the tax or the transfer

- A large regressive tax can be more equalizing than a small progressive one as shown in next slide
### Redistributive Effect and the Progressivity and Level of Taxes

<table>
<thead>
<tr>
<th></th>
<th>Gross Income</th>
<th>Tax A=50.5%</th>
<th>Net Income under A</th>
<th>Tax B=1%</th>
<th>Net Income under B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
<td>21%</td>
<td>1</td>
<td>2%</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>79%</td>
<td>50</td>
<td>98%</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>101</td>
<td>100%</td>
<td>51</td>
<td>100%</td>
<td>50</td>
</tr>
</tbody>
</table>

Source: Duclos and Tabi, 1996, Table 1.
FISCAL SYSTEM WITH MULTIPLE INTERVENTIONS
Fiscal Policy and Inequality
Three Key Questions

- Does the net fiscal system decrease inequality?
- Is a particular tax or transfer equalizing or unequalizing?
- What is the contribution of a particular tax or transfers (or any combination of them) to the change in inequality?
Does the net fiscal system decrease inequality?

Let’s define the Redistributive Effect of the net fiscal system as

$$RE_N = G_x - G_N$$

Where $G_x$ and $G_N$ are the Gini coefficient before and after the tax and the transfer, respectively.
Does the net fiscal system decrease inequality?

From Lambert (2001), we know that $RE_N$ is equal to the weighted sum of the redistributive effect of taxes and transfers

$$RE_N = \frac{(1 - g)RE_t + (1 + b)RE_B}{1 - g + b}$$

Where

- $RE_t$ and $RE_B$ are the Redistributive Effect of the tax and the transfer, respectively
- $g$ and $b$ are the tax and transfer level: i.e., total taxes and total transfers divided by total pre-tax and pre-transfer income, respectively
Does the net fiscal system decrease inequality?

For the net fiscal system to be equalizing:

\[ RE_N = \frac{(1-g)RE_t + (1+b)RE_B}{1-g+b} > 0 \]

Condition 1:

\[ \rightarrow RE_t > -\frac{(1+b)}{(1-g)} RE_B \]
Does the net fiscal system decrease inequality?

<table>
<thead>
<tr>
<th>Tax</th>
<th>Transfer</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regressive</td>
<td>Never Equalizing</td>
<td>Equalizing only if Condition 1 holds</td>
</tr>
<tr>
<td>Regressive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Progressive</td>
<td>Equalizing only if Condition 1 holds</td>
<td>Always Equalizing</td>
<td></td>
</tr>
</tbody>
</table>

**Condition 1:**

\[ \rightarrow R_E_t > -\frac{(1 + b)}{(1 - g)} R_E_B \]
Is a particular tax or transfer equalizing?

- If there is a single intervention in the system, any of the progressivity measures discussed earlier will give an unambiguous answer.

- If there is a tax and a transfer, then this is no longer the case.

  - A regressive tax can be equalizing and the reduction in inequality be larger with the tax than without it.
Lambert’s Conundrum

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original income x</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Tax Liability t(x)</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>42</td>
</tr>
<tr>
<td>Benefit level b(x)</td>
<td>21</td>
<td>14</td>
<td>7</td>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>Post-benefit income</td>
<td>31</td>
<td>34</td>
<td>37</td>
<td>40</td>
<td>142</td>
</tr>
<tr>
<td>Final income</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: Lambert, 2001, Table 11.1, P. 278
Lambert’s Conundrum

- The Redistributive Effect of the tax in this example is equal to -0.05, highlighting their regressivity.

- The Redistributive Effect of the transfer is equal to 0.19.

- Yet, the Redistributive Effect of the net fiscal system is 0.25, higher than the effect without the taxes!
Lambert’s Conundrum
Path Dependency

- If a tax is regressive vis-à-vis the original income but progressive with respect to the less unequally distributed post-transfer income

- Regressive taxes *can* exert an equalizing effect over an above the effect of progressive transfers
When could a regressive tax exert an equalizing force?

For the reduction in inequality to be higher with the tax than without it, the following condition must hold:

\[
RE_N = \frac{(1 - g)RE_t + (1 + b)RE_B}{1 - g + b} > RE_B
\]

Condition 2

\[
\rightarrow RE_t > -\frac{(g)}{(1 - g)} RE_B
\]
Is a tax equalizing?
Answer for a system with a tax and a transfer

<table>
<thead>
<tr>
<th>Adding a tax that is:</th>
<th>Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regressive</td>
</tr>
<tr>
<td>Tax</td>
<td>Never more equalizing</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Regressive</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Progressive</td>
<td>More equalizing only if</td>
</tr>
<tr>
<td></td>
<td>Condition 2 holds</td>
</tr>
</tbody>
</table>

Condition 2

\[ \rightarrow RE_t > - \frac{(g)}{(1 - g)} RE_B \]
Equalizing Regressive Taxes Exist in Real Life


- Chile’s 1996 fiscal system had equalizing regressive taxes (Engel et al., 1999)
  - Redistributive Effect of Net Fiscal System (taxes and transfers together = 0.0583 (decline in Gini points)
  - Redistributive Effect of System with Taxes only = - 0.0076
  - Redistributive Effect of System with Transfers but without Taxes = 0.0574

- Note that 0.0583 > 0.0574
Is a particular tax or transfer equalizing?

- Conditions to determine whether a transfer is equalizing are also available (in chapter but not presented here)

- The results shown above can be generalized to \( m \) taxes and \( n \) transfers (in chapter but not presented here)

- Note that the results do not depend on the tax and the transfer being of the same level (see conditions 1 and 2 above)
Path Dependency Underscores the Importance of Comprehensive Analysis

- **Obvious reason**
  - To capture the full effect of the net fiscal system

- **More subtle but fundamental reason**

  - Assessing the progressivity of a tax or a transfer in isolation can give the wrong answer to the question: Is the tax or the transfer equalizing?

  - Think of the example of Chile just shown above
How assessing the impact of a tax in isolation could give you the wrong answer

- Chile’s 1996 fiscal system had equalizing regressive taxes (Engel et al., 1999)
  - Redistributive Effect of Net Fiscal System (taxes and transfers together) = 0.0583 (decline in Gini points)
  - Redistributive Effect of System with Taxes only = -0.0076
  - Redistributive Effect of System with Transfers but without Taxes = 0.0574

If you focused on the effect of the tax in isolation, you would have concluded the tax is unequalizing since its Redistributive Effect is negative and equal to -0.0076

However, the regressive tax exerts an equalizing force when applied to the system with the transfers in place: 0.0583 > 0.0574
What is the contribution of a particular tax or transfer to the change in inequality?

- **Sequential method**
  - May give the wrong answer to the “with vs. without comparison” because it ignores path dependency

- **Marginal contribution method (same for poverty)**
  - Gives correct answer to the “with vs. without comparison” but does not fulfill the principle of aggregation: i.e., the sum of the marginal contributions will not equal the total change in inequality (except by coincidence)

- **Average Contribution with all paths considered (Shapley value)**
  - Fulfills the principle of aggregation, takes care of path dependency but the answer may be different from the marginal contribution => problematic
Calculating the Marginal Contribution of a Tax

The marginal contribution of a tax is defined as

$$MC_t = G_{x+B} - G_{x+B-t}$$

Where $G_{x+B-t}$ and $G_{x+B}$ are the Gini coefficient of incomes after the tax and the transfer and after the transfer only, respectively.

If $MC_t > 0$, remember, the tax is equalizing.
Sequential vs. Marginal Contribution
Why the sequential method can be misleading

- Chile’s 1996 fiscal system (Engel et al., 1999)
  
  - Redistributive Effect of Net Fiscal System (taxes and transfers together) = 0.0583 (decline in Gini points)
  - Redistributive Effect of System with Taxes only = -0.0076
  - Redistributive Effect of System with Transfers but without Taxes = 0.0574
Sequential vs. Marginal Contribution
Why the sequential method can be misleading

Sequential contribution method

- If you calculated the contribution of taxes to the change in inequality by subtracting the Gini after taxes from the Gini pre-tax-pre-transfers, you would have concluded that the contribution of taxes was unequalizing to the tune of -0.0076

which is inconsistent with the fact that if you take the taxes out, the reduction in inequality is smaller
Sequential vs. Marginal Contribution
Why the sequential method can be misleading

Marginal contribution method

- The marginal contribution of adding the tax to the system with the transfer in place is equal to the difference of the Redistributive Effect of the net fiscal system and the Redistributive Effect of the system without the taxes (with transfers only)

\[ 0.0583 - 0.0574 = 0.009 \]

A positive value which is consistent with how adding the tax causes inequality to fall
Relaxing Assumptions

- **Reranking**: individuals can swap positions in the post-fiscal income ordering; true of all systems in the real world

- **No dominance**: post-fiscal Lorenz curve crosses the pre-fiscal Lorenz curve; normative parameter must be explicitly introduced (will not be covered today)

- **Different pre-fiscal (original) distributions**: comparing the inequality- and poverty-reducing capacity of fiscal systems across countries and over time (will not be covered today)
Reranking

- Reranking diminishes the redistributive capacity of fiscal policy
- Think of the following extreme example
  - The fiscal system only causes individuals to swap places but the incomes of poorest, second poorest, up to the richest individual stay the same
  - Post-fiscal inequality after taxes and transfers will remain unchanged
  - Fiscal policy only produced a lot of “churning”
Estimating the Effect of Reranking

The Redistributive Effect (Gini for income before taxes and transfers minus Gini for income after taxes and transfers) can be written as

$$RE_N = (G_x) - (G_N)$$

By adding and subtracting $CC^X_N$, we can rewrite the Redistributive Effect as:

$$RE_N = (G_x - CC^X_N) - (G_N - CC^X_N)$$

where $CC^X_N$ is the concentration coefficient for income after taxes and transfers
Estimating the Effect of Reranking

Then, the Redistributive Effect can be written as:

\[ RE_N = VE - RR \]

where:

- **VE**, the vertical equity component, is known as the Reynolds-Smolensky Index. If there is no re-ranking, \( RE = VE \) by definition because the concentration coefficient for income after taxes and transfers will be identical to the Gini coefficient for income after taxes and transfers.

- **RR**, the reranking component, is known as the Atkinson-Plotnick index of horizontal inequity. If there is no reranking, this term will equal zero.

Thus, **RR** can be calculated as:

\[ RR = VE - RE_N \]
How important is reranking in actual fiscal systems?

- In some countries, the reranking effect can be huge.
- For example, in Bolivia the redistributive effect before in-kind transfers is zero. The fiscal system only induced reranking

<table>
<thead>
<tr>
<th></th>
<th>South Africa</th>
<th>Bolivia</th>
<th>Brazil</th>
<th>Indonesia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini (Market income)</td>
<td>0.771</td>
<td>0.503</td>
<td>0.579</td>
<td>0.418</td>
</tr>
<tr>
<td>Gini (Post-fiscal income)</td>
<td>0.695</td>
<td>0.503</td>
<td>0.546</td>
<td>0.416</td>
</tr>
<tr>
<td>Redistributive Effect¹</td>
<td>0.077</td>
<td>0.000</td>
<td>0.033</td>
<td>0.002</td>
</tr>
<tr>
<td>Vertical Equity (VE)²</td>
<td>0.083</td>
<td>0.003</td>
<td>0.048</td>
<td>0.007</td>
</tr>
<tr>
<td>Reranking Effect (RR)³</td>
<td>0.006</td>
<td>0.003</td>
<td>0.014</td>
<td>0.005</td>
</tr>
<tr>
<td>RR/VE</td>
<td>0.075</td>
<td>1.000</td>
<td>0.300</td>
<td>0.706</td>
</tr>
</tbody>
</table>

If there is reranking, conditions 1 and 2 discussed above apply to the vertical equity (VE) component of

\[ RR = VE - RE_N \]
Comparing Impact of Fiscal Systems Across Countries and Over Time

- Determining when a fiscal intervention or a system is more equalizing than another in cross-country and over-time comparisons involves comparing cases with different pre-tax-pre-transfer income distributions.

- Two methods have been proposed:
  - Select a country or a time period as baseline
  - “Transplant and compare” method (Dardanoni and Lambert, 2000)
Poverty Impact

- Determining when a fiscal intervention is poverty-reducing
  - Compare standard poverty measures using the marginal contribution approach

- Fiscal policy can increase poverty to the point that it is left higher than before taxes and transfers
  - Showed in Session 1 that we found this in five out of thirteen countries in CEQ
Indirect Taxes increase poverty over and above market income poverty in 5 cases

Change in Headcount Ratio ($2.5 PPP/Day)
(in percentage points)

Poverty Impact

- A tax system can be equalizing but poverty-increasing and poverty can end up above what prevailed before fiscal policy

  - Example Ethiopia

  - Do not use word “regressive” for a poverty increasing intervention
Note that Net Indirect Taxes can be equalizing and yet poverty increasing: Ethiopia

Change in Headcount Ratio (\$2.5 PPP/Day): Marginal Contribution from Net Indirect Taxes
(in percentage points)

Change in Gini: Marginal Contribution of Net Indirect Taxes
(in GINI points)
Poverty Impact

- Even if poverty measures do not increase, the poor can be made poorer by the fiscal system and some of the nonpoor can be made poor.

- In Brazil, more than a third of the pre-fiscal policy poor are made poorer by fiscal policy (excluding transfers in-kind, of course).

- Fiscal Impoverishment Index
  
Main messages

- To determine whether a fiscal intervention is equalizing or not, one must assess its contribution with the other interventions in place.
  - A regressive tax, for example, can exert an equalizing force that is over and above a system without that regressive tax.

- To measure the size of the contribution, use the marginal contribution method but remember that adding the marginal contributions will not be equal to the total change.

- The impact of a tax on inequality and poverty can go in opposite directions: e.g., equalizing and poverty increasing.

- An important proportion of the poor may be left poorer (in cash) by the fiscal system, and current measures may not alert us to this: new measure of *fiscal impoverishment* does
Readings


Additional Readings

Thank you!